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THE SINE RULE

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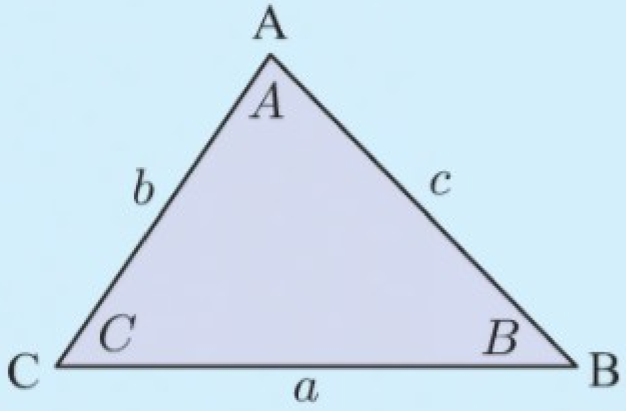
In any triangle ABC with sides a, b, and c units, and opposite angles A, B, and C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



Proof:

The area of any triangle ABC is given by $\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$.

Dividing each expression by $\frac{1}{2}abc$ gives $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

GEOMETRY
PACKAGE

FINDING SIDES

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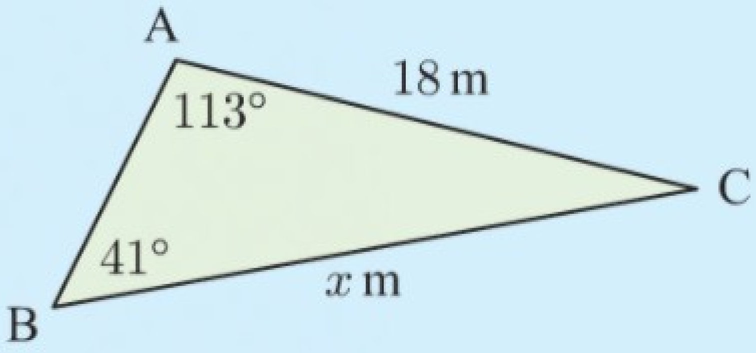
Example 13

Self Tutor

Find x, rounded to 2 decimal places:



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Example 13 **Self Tutor**

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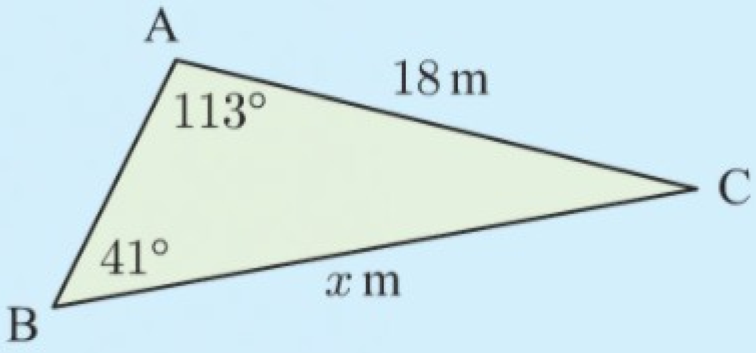
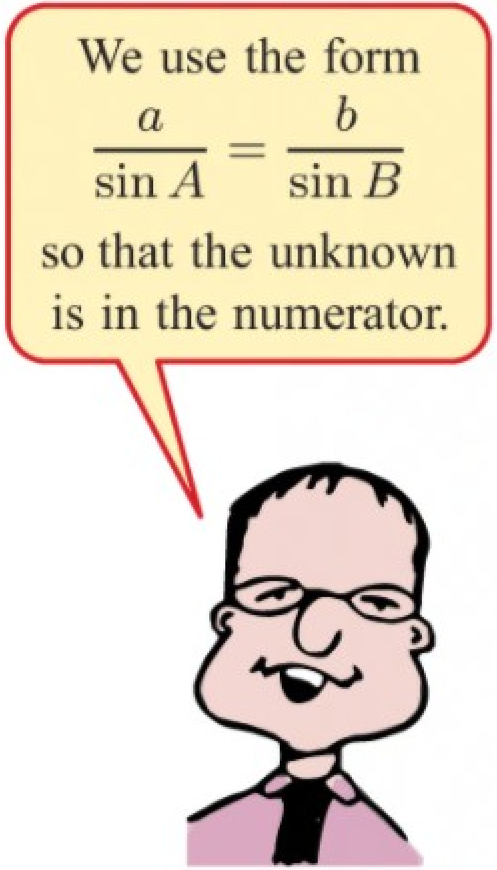


Using the sine rule, $$\frac{x}{\sin 113°} = \frac{18}{\sin 41°}$$

$$\therefore \quad x = \frac{18 \times \sin 113°}{\sin 41°}$$

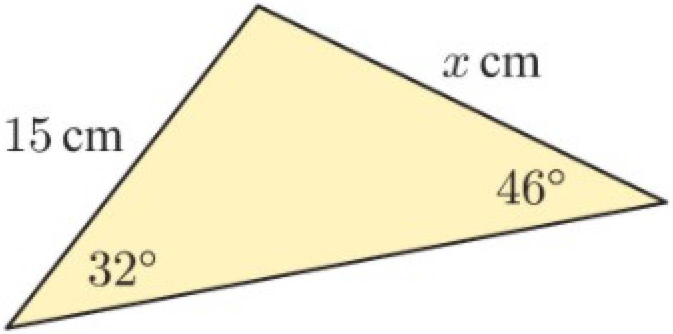
$$\therefore \quad x \approx 25.26$$


We use the form $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ so that the unknown is in the numerator.

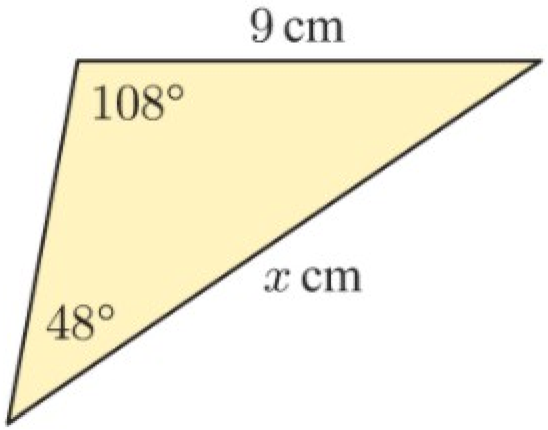
EXERCISE 12G.1

1 Find x, rounded to 2 decimal places:

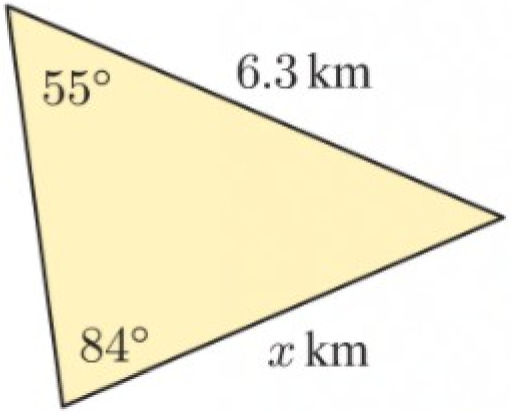
a



b

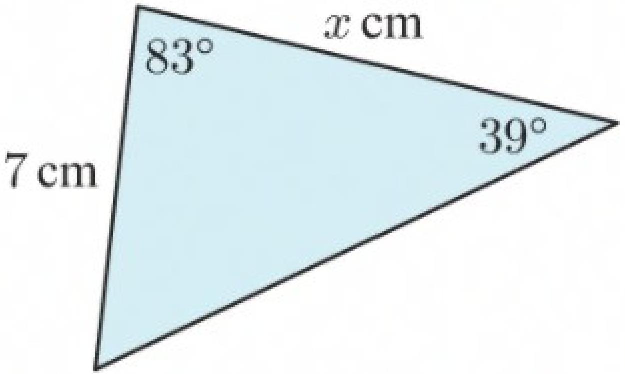


c



2 Find x:

a



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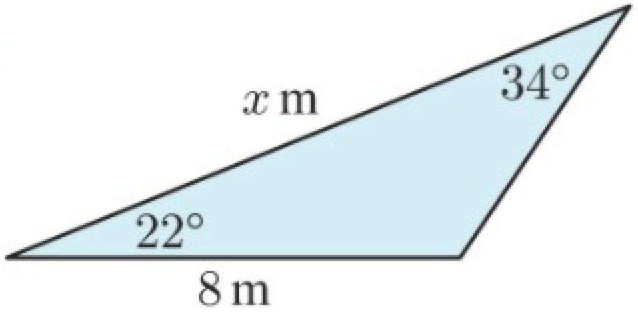

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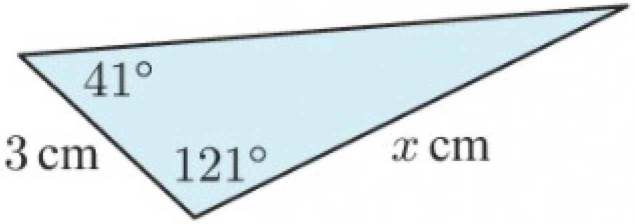
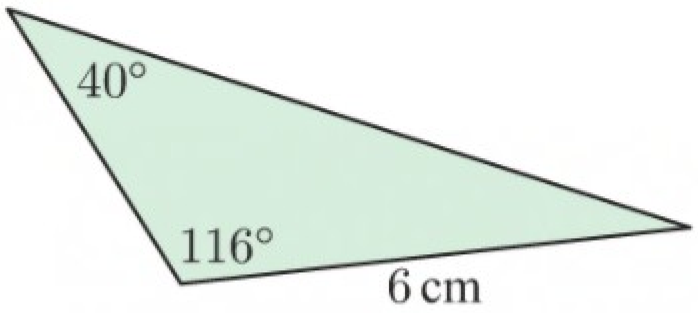
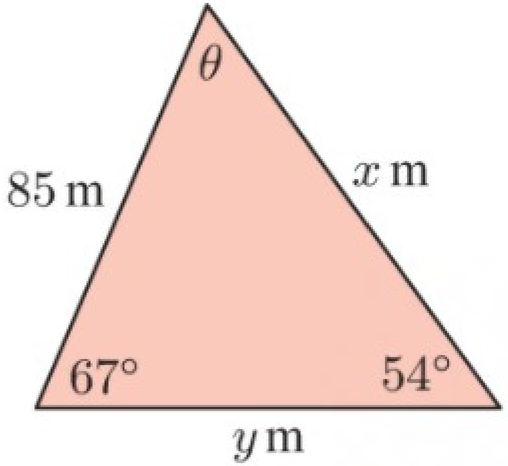


3 Find the area of this triangle.

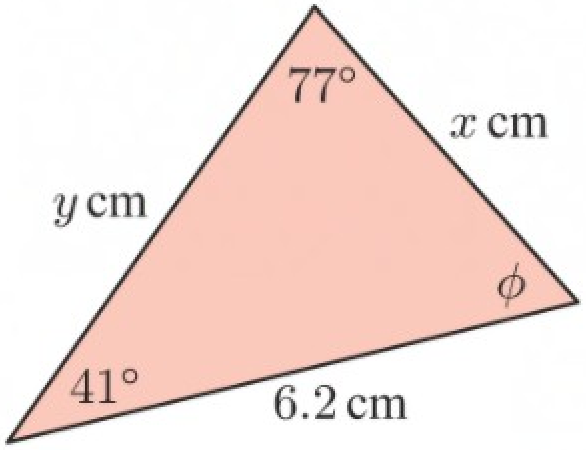


4 Find *all* unknown sides and angles of:

a



b



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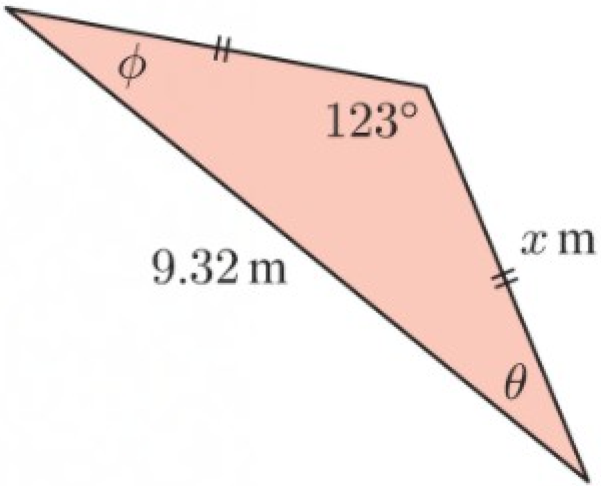

FINDING ANGLES

Finding angles using the sine rule is more complicated than finding sides because there may be two possible answers.

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$$\sin(180^\circ - \theta) = \sin\theta.$$

Example 14 **Self Tutor**

In triangle ABC, AB $= 12$ cm, AC $= 8$ cm, and angle B measures $28°$.
Find, rounded to 1 decimal place, the measure of angle C.

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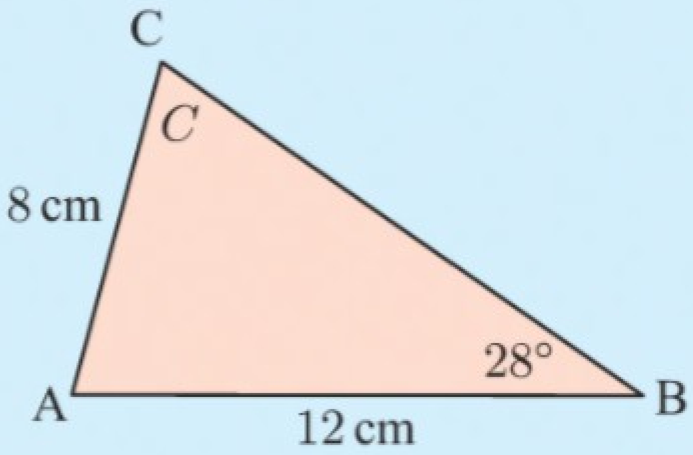
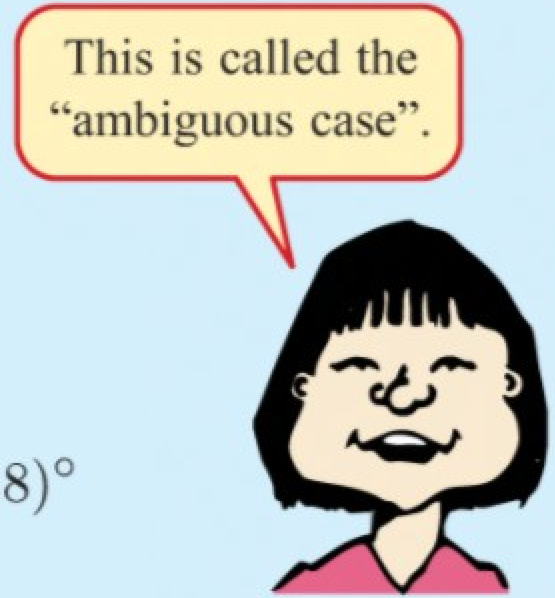


$$\frac{\sin C}{12} = \frac{\sin 28^\circ}{8} \quad \{\text{sine rule}\}$$

$$\therefore \quad \sin C = \frac{12 \times \sin 28^\circ}{8}$$

$$\text{Now} \quad \sin^{-1}\left(\frac{12 \times \sin 28^\circ}{8}\right) \approx 44.8^\circ$$


This is called the "ambiguous case".

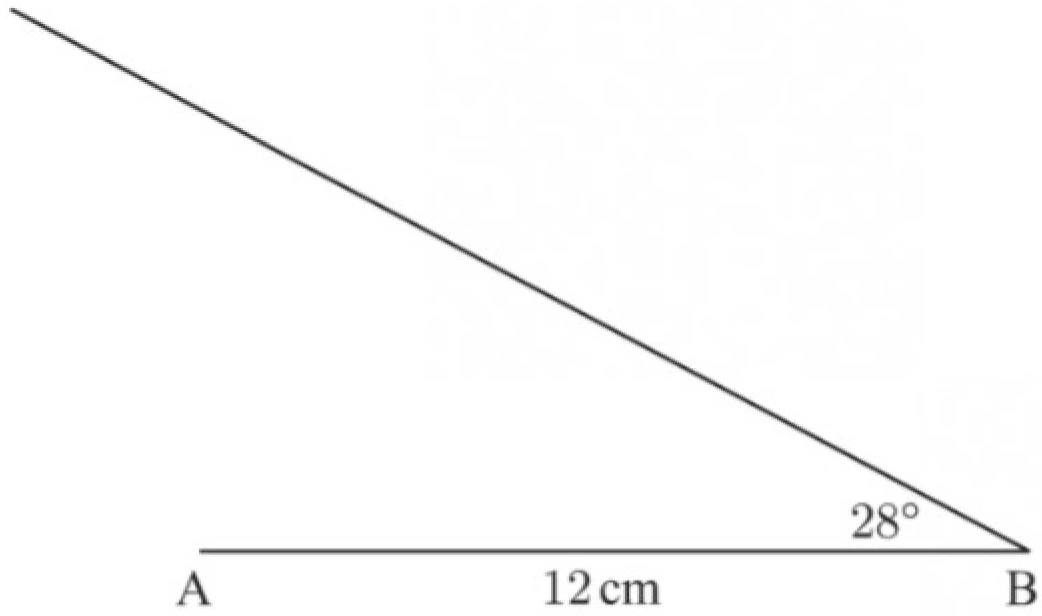
Since the angle at C could be acute or obtuse, $C \approx 44.8^\circ$ or $(180 - 44.8)^\circ$

$$\therefore \quad C \approx 44.8^\circ \text{ or } 135.2^\circ$$

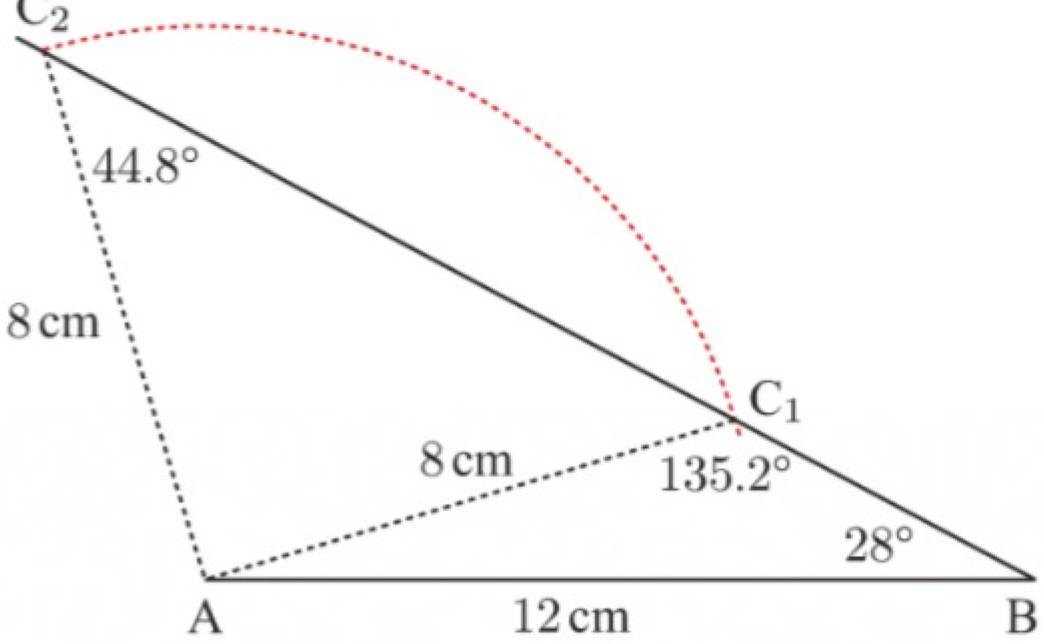
In this case there is insufficient information to determine the actual shape of the triangle.

The validity of the two answers in the above **Example** can be demonstrated by a simple construction.

Step 1: Draw [AB] of length 12 cm, and construct an angle of 28° at B.



Step 2: From A, draw an arc of radius 8 cm.



Example 15 **Self Tutor**

In triangle KLM, $L\widehat{K}M$ measures $52°$, $LM = 158$ m, and $KM = 128$ m.
Find the measure of angle L.

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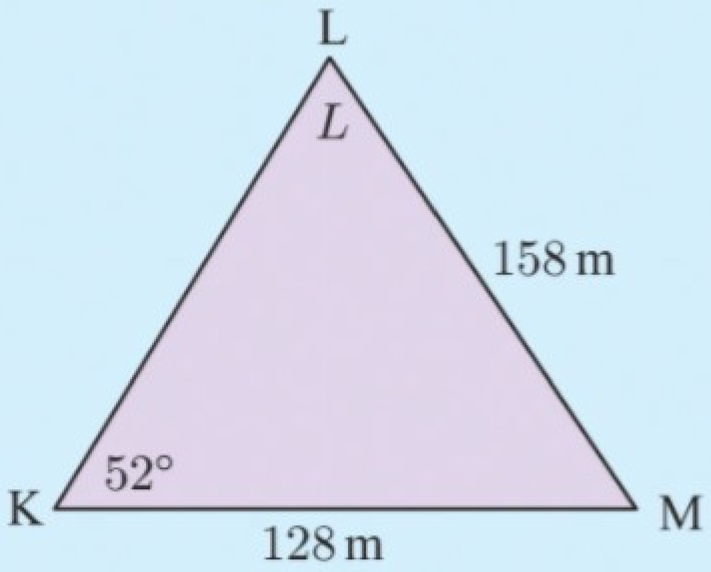
In triangle KLM, $L\widehat{K}M$ measures 52°, $LM = 158$ m, and $KM = 128$ m.
Find the measure of angle L.



$$\frac{\sin L}{128} = \frac{\sin 52^\circ}{158} \quad \{\text{sine rule}\}$$

$$\therefore \quad \sin L = \frac{128 \times \sin 52^\circ}{158}$$

$$\text{Now} \quad \sin^{-1}\left(\frac{128 \times \sin 52^\circ}{158}\right) \approx 39.7^\circ$$

$\therefore$ since L could be acute or obtuse,

$$L \approx 39.7^\circ \quad \text{or} \quad (180 - 39.7)^\circ \approx 140.3^\circ$$

However, we can reject $L \approx 140.3^\circ$ as $140.3^\circ + 52^\circ > 180^\circ$ which is impossible.

$\therefore$ the angle $L \approx 39.7^\circ$.

EXERCISE 12G.2

1 In triangle ABC, $AB = 10$ cm, $BC = 7$ cm, and $C\widehat{A}B$ measures 42°.

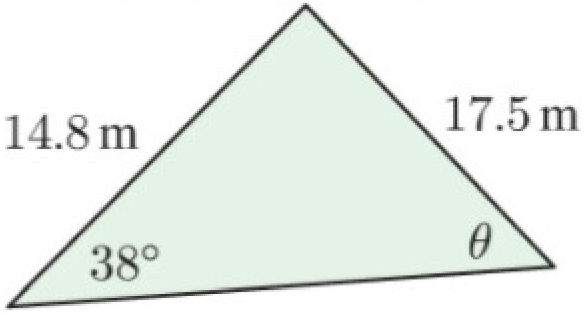
- **a** Find the two possible values for $A\widehat{C}B$.
- **b** Draw a diagram to illustrate the two possible triangles.

2 In triangle PQR, $PQ = 10$ cm, $QR = 12$ cm, and $R\widehat{P}Q$ measures 42°.

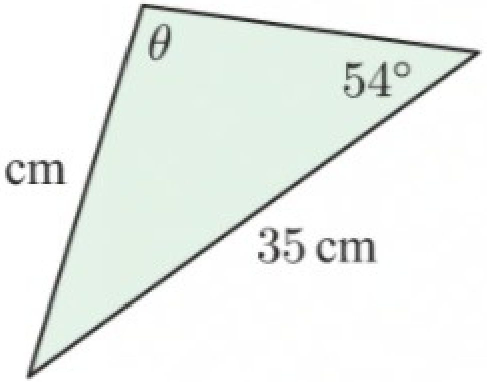
- **a** Show that there is only one possible value for $P\widehat{R}Q$, and state its measure.
- **b** Draw a diagram to demonstrate that only one triangle can be drawn from the information given.

3 The following diagrams are not drawn to scale, but the information on them is correct. Find the value of θ:

a

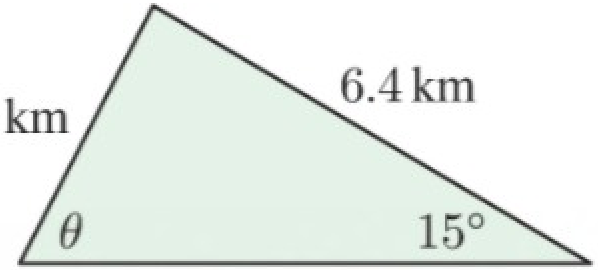


b

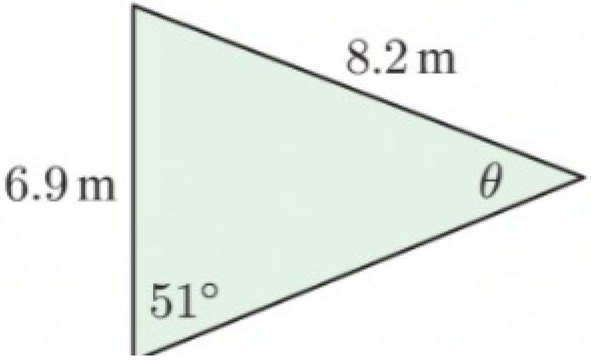


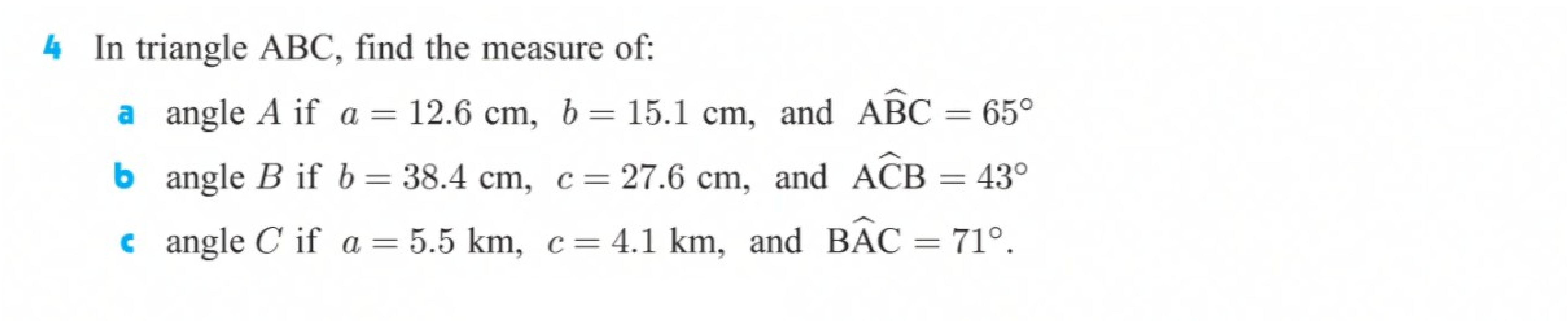
There may be two possible solutions.

c



d





4 In triangle ABC, find the measure of:

- **a** angle A if $a = 12.6$ cm, $b = 15.1$ cm, and $A\widehat{B}C = 65^\circ$
- **b** angle B if $b = 38.4$ cm, $c = 27.6$ cm, and $A\widehat{C}B = 43^\circ$
- **c** angle C if $a = 5.5$ km, $c = 4.1$ km, and $B\widehat{A}C = 71^\circ$.

EXERCISE 12G.1

1 **a** $x \approx 11.05$ **b** $x \approx 11.52$ **c** $x \approx 5.19$

2 **a** $x \approx 9.43$ **b** $x \approx 11.9$ **c** $x \approx 6.37$

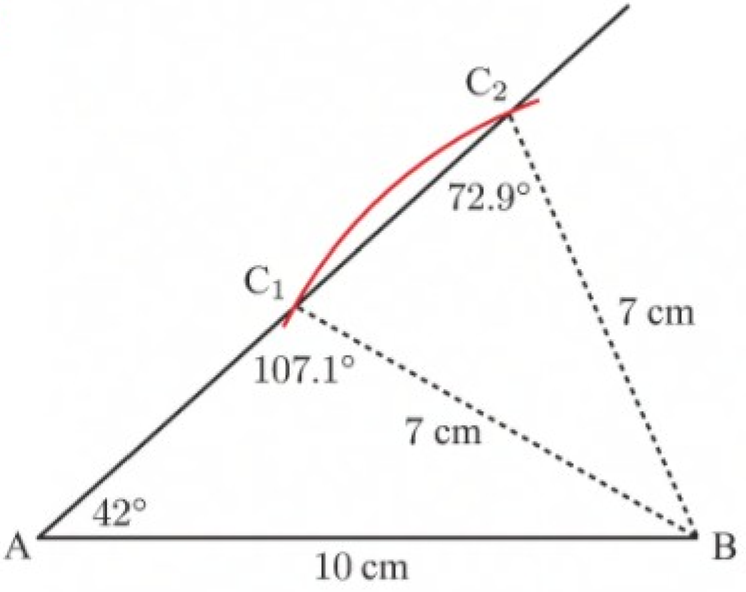
3 $\approx 10.2 \text{ cm}^2$

4 **a** $\theta = 59°, \quad x \approx 96.7, \quad y \approx 90.1$
b $\phi = 62°, \quad x \approx 4.17, \quad y \approx 5.62$
c $\theta = \phi = 28.5°, \quad x \approx 5.30$

EXERCISE 12G.2

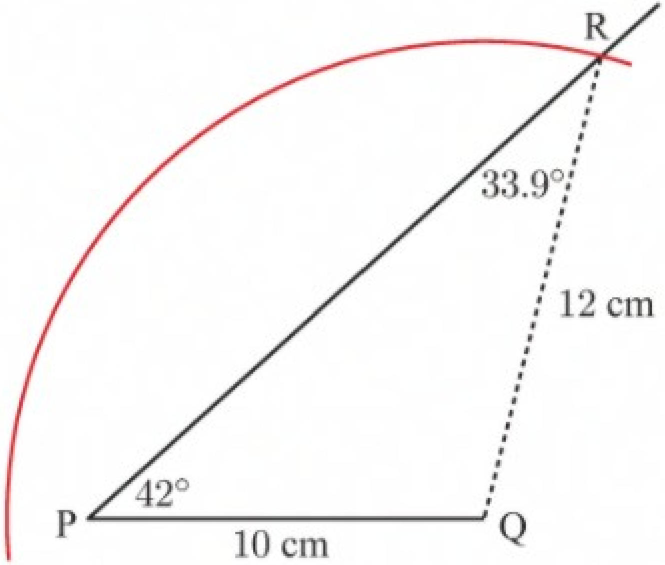
1 **a** $\widehat{ACB} \approx 72.9°$
or $\approx 107°$

b



2 **a** Using the sine rule, $\widehat{PRQ} \approx 33.9°$ or $146.1°$.
In the case of $146.1°$, $42° + 146.1°$ is already $> 180°$.
$\therefore$ $\widehat{PRQ} \approx 33.9°$ only.

b



3 **a** $\theta \approx 31.4°$ **b** $\theta \approx 77.5°$ or $102°$
c $\theta \approx 43.6°$ or $136°$ **d** $\theta \approx 40.8°$

4 **a** $\widehat{A} \approx 49.1°$ **b** $\widehat{B} \approx 71.6°$ or $108°$ **c** $\widehat{C} \approx 44.8°$